PREREQ. PRACTICE PROBS 1 SOLUTIONS

1) $\frac{1}{2}(x-2y)^2 - x^2 - \frac{3}{2}x\left(y-\frac{1}{2}\right)$

$= \frac{1}{2}(x^2 - 4xy + 4y^2) - x^2 - \frac{3}{2}xy + \frac{3}{4}x$

$= \frac{1}{2}x^2 - 2xy + 2y^2 - x^2 - \frac{3}{2}xy + \frac{3}{4}x$

$= -\frac{1}{2}x^2 - \frac{7}{2}xy + 2y^2 + \frac{3}{4}x$.

2) $\left(\frac{2}{3}x-3\right)\left(\frac{2}{3}x+3\right) - (1-x)(1+x)^2$

$= \left(\frac{4}{9}x^2 - 9\right) - (1-x^2)(1+x)$

$= \frac{4}{9}x^2 - 9 - (1 + x - x^2 - x^3)$

$= \frac{4}{9}x^2 - 9 - 1 - x + x^2 + x^3$

$= x^3 + \frac{13}{9}x^2 - x - 10$

3) $\dfrac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x^2}} = \dfrac{\frac{x-(x+1)}{x(x+1)}}{\frac{1}{x^2}} = \dfrac{\frac{-1}{x(x+1)}}{\frac{1}{x^2}} = -\frac{1}{x(x+1)} \cdot x^2 = \frac{-x}{x+1}$

4) $$\frac{x^3+x^2+x+1}{x^2-1} - \frac{9x^2+6x+1}{3x^2-3x+x-1} + \frac{2}{x}$$

$$= \frac{x^2(x+1)+(x+1)}{(x+1)(x-1)} - \frac{(3x+1)^2}{3x(x-1)+(x-1)} + \frac{2}{x}$$

$$= \frac{\cancel{(x+1)}(x^2+1)}{\cancel{(x+1)}(x-1)} - \frac{(3x+1)^{\cancel{2}}}{(x-1)\cancel{(3x+1)}} + \frac{2}{x}$$

$$= \frac{x^2+1}{x-1} - \frac{3x+1}{x-1} + \frac{2}{x}$$

$$= \frac{(x^2+1)x - (3x+1)x + 2(x-1)}{x(x-1)}$$

$$= \frac{x^3+\cancel{x}-3x^2-\cancel{x}+2x-2}{x(x-1)}$$

$$= \frac{x^3-3x^2+2x-2}{x(x-1)}$$

5) $16x^3+8x^2+x = x(16x^2+8x+1) = x(4x+1)^2$

6) $x^6-\frac{1}{9}x^2 = x^2\left(x^4-\frac{1}{9}\right) = x^2\left(x^2+\frac{1}{3}\right)\left(x^2-\frac{1}{3}\right)$

$= x^2\left(x^2+\frac{1}{3}\right)\left(x+\sqrt{\tfrac{1}{3}}\right)\left(x-\sqrt{\tfrac{1}{3}}\right)$

7) $3y^3+2y^2-y = y(3y^2+2y-1) = y\left(3(y+1)\left(y-\frac{1}{3}\right)\right)$

$\boxed{y_1, y_2 = -1,\ \frac{1}{3}}$ $= y(y+1)(3y-1)$

8) $2x^3-2+2x-2x^2 = 2x^2(x-1)+2(x-1) = (x-1)(2x^2+2)$

$= 2(x-1)(x^2+1)$

9*) $x^4+4 = \underbrace{x^4+4+4x^2} -4x^2$

$= (x^2+2)^2 - 4x^2$

$= (x^2+2+2x)(x^2+2-2x)$

10) $\frac{2}{3}(x-1)+2x = 1+\frac{8}{3}x$

$\frac{2}{3}x - \frac{2}{3} + 2x = 1 + \frac{8}{3}x$

$\cancel{\frac{8}{3}x} - \frac{2}{3} = 1 + \cancel{\frac{8}{3}x}$

$-\frac{2}{3} = 1$ $\boxed{\nexists x \in \mathbb{R} \text{ (impossible)}}$

11) $3x^3 - 4x^2 - x = 0$

$x(3x^2-4x-1)=0$

$x = 0$

$3x^2-4x-1=0$

$x_1, x_2 = \frac{4 \pm \sqrt{16+12}}{6} = \frac{4 \pm \sqrt{28}}{6}$

$= \frac{4 \pm 2\sqrt{7}}{6} = \frac{2 \pm \sqrt{7}}{3}$

3 solutions $\boxed{x_1 = \frac{2-\sqrt{7}}{3}, \ x_2 = \frac{2+\sqrt{7}}{3}, \ x_3 = 0.}$

12) $\frac{x}{1-x} - \frac{1}{2} = \frac{2-x}{x}$

Domain $x \neq 1$
$x \neq 0$

Multiply by $2x(1-x)$:

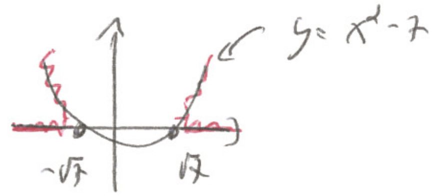
$2x^2 - x(1-x) = (2-x)2(1-x)$

$\cancel{2x^2} - x + x^2 = 4 - 6x + \cancel{2x^2}$

$x^2+5x-4=0$ $\boxed{x_1, x_2 = \frac{-5 \pm \sqrt{41}}{2} \text{ acceptable}}$

13) $\sqrt{x^2-7} = 3$

Domain $x^2-7 \geq 0 \rightarrow \boxed{x \leq -\sqrt{7} \cup x \geq \sqrt{7}}$



(because $x^2-7=0 \quad x = \pm\sqrt{7}$)

$x^2-7 = 9$

$x^2 = 16$

$\boxed{x = \pm 4 \quad \text{acceptable}}$ because $-4 \leq -\sqrt{7}$

$4 \geq \sqrt{7}$

14) $x^5+1=0$

$x^5 = -1$

$\boxed{x=-1}$

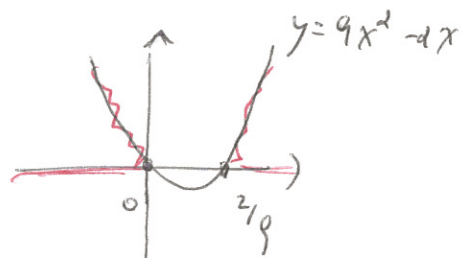
15) $\dfrac{3-x}{9x^2-2x} \geq 0$

$N \geq 0 \quad 3-x \geq 0 \rightarrow x < 3$

$D > 0 \quad 9x^2-2x \geq 0$

$9x^2-2x = 0 \rightarrow x=0$

$\rightarrow x = \frac{2}{9}$



$x < 0 \cup x > 2/9$

	0	$2/9$	3	
N	+	+	+	−
D	+	−	+	+
Q	+	−	+	−

$\boxed{x < 0 \cup 2/9 < x < 3}$